Given a graph $G$, a perfect matching of $G$ is a matching which covers all the vertices (equivalently, a matching of cardinality $|V|/2$).

Suppose you are given an oracle that, given a graph $G$, tells you whether $G$ has a perfect matching or not. Show how to use this oracle to determine the maximum cardinality matching of a graph $G(V,E)$. The total number of calls to the oracle (to find the cardinality of the maximum matching, and then to find the matching itself) should be at most $|V| + |E|$. 

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1. You are allowed to submit your solutions in groups of at most three students.