Let $P = \{ x \in \mathbb{R}^n : Ax \leq b \}$ be a bounded, non-empty set. Formulate a linear program that computes the radius of the largest ball that can be inscribed into $P$.

**Sol.** A ball of radius $r$ and center $x$ is contained in $P$ if and only if $x \in P$ and $x$ has distance at least $r$ from any hyperplane defining $P$. Hence we obtain the following linear program:

$$
\begin{align*}
\max & \quad r \\
\text{s.t.} & \quad \frac{b_i - ax}{\|a_i\|} \geq r & \forall i = 1, \ldots, m \\
& \quad Ax \leq b
\end{align*}
$$

where $a_1, \ldots, a_m$ are the rows of $A$ and $b = (b_1 \ldots b_m)^\top$.