PhD Doctoral Course – Network Design – 29th September 2009

3rd Assignment

1. Show that, given an instance of the Steiner Tree problem, you can assume w.l.o.g. that the input graph is a metric graph.

2. Show that, given an instance of the Steiner Tree problem on a metric graph, the number of Steiner nodes in an optimal solution is $\leq |R|-2$, where R is the set of terminals to be connected.

3. Give an exact algorithm for the Steiner Tree problem that runs in polynomial time if $|R|$ is a *fixed* parameter (e.g. $|R|=5$).

4. The aim of this exercise is to describe a bit-scaling technique that can be employed to derive an implementation of the 3/2-approximation algorithm for Steiner Tree problem on quasi-bipartite graphs $G(R \cup X, E)$, with running time polynomial in the size of the input [R. Rizzi 1999].
Consider the sequence of costs $c = c_0, c_1, \ldots, c_k$ where, for $i > 0$, $c_i(e) := \lfloor \frac{c_{i-1}(e)}{2} \rfloor$.
Let k be the smallest index for which $c_k(e) \leq 1$ for every edge e. Note that $k \leq \log_2(\max\{c(e) : e \in E\})$.
Still, for $i = 0, 1, \ldots, k$ let T_i^* be an optimal Steiner tree and T_i be a $\frac{3}{2}$-optimal Steiner tree in $G(R \cup X, E)$ with cost vector c_i.
When the algorithm is executed on $G(R \cup X, E)$ with costs c_k as input, then the "while loop" will cycle at most n times, since c_k is a $0,1$-vector. The output will be a $\frac{3}{2}$-optimal Steiner tree T_k. Show that:

- $c_{i-1}(T_i) \leq \frac{3}{2}n + \frac{3}{2}c_{i-1}(T_{i-1}^*)$.
- Using the previous result, show how to output a $\frac{3}{2}$-optimal Steiner tree for the original instance in polynomial time.