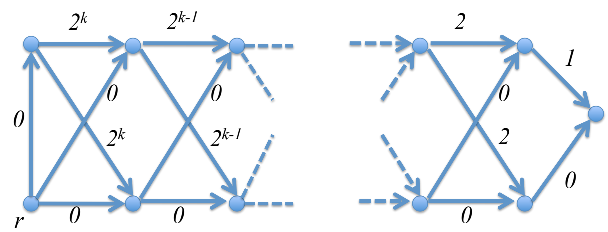
PhD Doctoral Course – Network Design – 22th September 2009

2nd Assignment

1. Consider the graph G_k reported in the following figure. Show that Ford's algorithm can take more than 2^k steps to solve the shortest path problem on G_k.



2. We are given numbers $a_1, \ldots, a_n$. We want to find i and j, with $1 \leq i \leq j \leq n+1$ so that $\sum_{k=i}^{j-1} a_k$ is minimized. Reduce the problem to a shortest path problem.

3. Give an example to show that Dijkstra's algorithm can give an incorrect result if negative costs are allowed.

4. Let $G(V,E)$ be a directed connected graph with cost vector c such that there are no negative-cost directed cycles. Let $r, s \in V$. Prove that:

$$\min\{c(P) : P \text{ is a path from } r \text{ to } s\ \} = \max\{y_s : y \text{ is a feasible potential.}\}$$

5. Show that the maximization above is equivalent to the following linear programming problem $\mathcal{P}$:

$$\max\ y_s - y_r$$
$$y_w - y_v \leq c_{vw} \quad \forall\, vw \in E$$

6. Write the dual $\mathcal{D}$ of $\mathcal{P}$, and show that:

- any path from r to s provides a feasible solution to $\mathcal{D}$.
- $\mathcal{D}$ has an optimal solution that is the characteristic vector of a simple path.