Prof. Friedrich Eisenbrand
Location: MA A3 31

Due date: 20.11.09

Practical exercises

Optimization Methods in Finance

Fall 2009

In this exercise you have to implement the Frank-Wolfe algorithm for convex optimization and to apply it for Mean Variance Optimization.

1. *Implement the Frank-Wolfe algorithm in C++ (or Matlab).* More precisely, you should implement a method

```
Vector<double> runFrankWolfe(Matrix<double> Q, Matrix<double> A,
                             Vector<double> b, int K)
```

   that applies the Frank-Wolfe algorithm to the system

   $$\min\{x^T Qx \mid Ax = b, x \geq \mathbf{0}\}$$

   for K many iterations. The method should return the solution x.
   Hints:

   - We recommend you to test the algorithm on a small 2-dimensional example to check its correctness.
   - For the linear optimization step in the algorithm, you can use an LP solving library. More in the section *Solving LPs within C++*.

2. *Solve a Mean-Variance Optimization problem with your algorithm.*

   - You can find historical asset prices either in the R-file (R is a statistics program)

http://disopt.epfl.ch/webdav/site/disopt/shared/OptInFinance09/swx_histo_swissquote.dat

   or in the corresponding Microsoft Excel file

http://disopt.epfl.ch/webdav/site/disopt/shared/OptInFinance09/swx_histo_swissquote.xls

   The numbers are weekly stock values (447 weeks, starting from 16.03.98).

   - Then you should choose a suitable number (say $n = 8$) of candidate assets (remove the rest). Compute the average return vector r and covariance matrix Q using the formulas from the lecture (see slides from 14th of October). Recall that if $I_{i,t}$ is the price of asset i at the beginning of week $t = 0, \dots, T$ $(T = 446)$, then

$$\begin{array}{rcll}
r_{it} & := & \frac{I_{i,t} - I_{i,t-1}}{I_{i,t-1}} & \text{return of asset } i \text{ in week } t \\
\bar{r}_i & := & \frac{1}{T}\sum_{i=1}^{T} r_{it} & \text{(arithmetic) average return for asset } i \\
\mu_i & := & \left(\prod_{t=1}^{T}(1 + r_{it})\right)^{1/T} - 1 & \text{(geometric average) return} \\
\mathrm{cov}(i,j) & := & \frac{1}{T}\sum_{t=1}^{T}(r_{it} - \bar{r}_i)(r_{jt} - \bar{r}_j) & \text{covariance of assets } i \text{ and } j \\
\sigma_i & := & \sqrt{\mathrm{cov}(i,i)} & \text{volatility of asset } i \\
Q_{ij} & := & \frac{\mathrm{cov}(i,j)}{\sigma_i \cdot \sigma_j} & \text{correlation matrix}
\end{array}$$

- Solve the MVO problem

$$\begin{aligned} \min\ x^T Qx & \\ \sum_{i=1}^{n} x_i\mu_i &\geq R \\ \sum_{i=1}^{n} x_i &= 1 \\ x &\geq \mathbf{0} \end{aligned}$$

  using your Frank-Wolfe algorithm for a suitable choice of the (weekly) total return R (and a suitable number of iterations K).
  Hint: The book ”Optimization Methods in Finance” (by Cornuejols and Tütüncü, page 143) contains a well descripted example for MVO.

- Use Matlab (more precisely the command `quadprog`) to solve the same quadratic program.

1 Solving LPs within C++

For this programming exercise, you can use any LP-solver that is available for C++. We here present 2 possible choices:

- **LP-Solve:** This library can be downloaded following the link

  http://lpsolve.sourceforge.net/5.5/

  The website provides a documentation and an example

  http://lpsolve.sourceforge.net/5.5/formulate.htm#C/C++

  You can test whether you installed the library correctly if you create a file `demo.cpp` with the content

```
#include <stdio.h>
#include "lp_lib.h"

main()
{
  lprec *lp;
  lp = make_lp(0,4);
  /* ... */
  delete_lp(lp);
}
```

  Then try to compile it with

```
g++ demo.cpp -I librarypath -L librarypath -llpsolve55
```

- **Soplex:** This library is already installed on the terminal PCs. A documentation can be found under

  `http://soplex.zib.de/soplex.shtml`

2 The submission

The submission must contain the following

- Your compilable C++ (or Matlab) code with the Frank-Wolfe algorithm
- The names of the candidate assets together with the computed portfolio x and the objective function value $x^T Qx$.
- The Matlab sheet where you call `quadprog`.

Please send the submission (and possible questions) till **20.11.09** to yanick.raja@epfl.ch.