

Practical exercises
Optimization Methods in Finance

Fall 2009

Practical exercise 3

The goal in this exercise is to detect arbitrage opportunities using the modelling language ZIMPL. Suppose we are given a set I of assets, a set J of scenarios, a vector S_0 of initial prices of the assets and a matrix S_1 , where the entry in the i th row and j th column denotes the price $S_0^i(j)$ of asset i at time 1 in scenario j . Moreover we have a risk-less asset 0 with return R . In ZIMPL syntax we consider the following two examples

Example 1:

```
set I := { 0 .. 5 };
set J := { 1 .. 8 };
param R := 1.05;
param S0[I] := <0>1.00, <1>1.10, <2>1.04, <3>0.91, <4>1.32, <5>0.75;
param S1[I*J] :=
|      1,      2,      3,      4,      5,      6,      7,      8 |
|0| 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05 |
|1| 0.98, 0.79, 0.97, 0.65, 1.03, 1.41, 1.11, 0.69 |
|2| 1.27, 1.27, 1.23, 1.12, 1.45, 0.94, 1.34, 1.25 |
|3| 0.70, 0.93, 1.38, 0.74, 0.97, 1.02, 1.10, 0.53 |
|4| 1.42, 0.73, 0.95, 0.95, 0.87, 1.37, 0.52, 1.12 |
|5| 1.03, 0.88, 0.85, 1.10, 1.25, 1.05, 0.80, 1.48 |;
```

Example 2:

```
set I := { 0 .. 2 };
set J := { 1 .. 8 };
param R := 1.05;
param S0[I] := <0>1.00, <1>0.84, <2>1.11;
param S1[I*J] :=
|      1,      2,      3,      4,      5,      6,      7,      8 |
|0| 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05, 1.05 |
|1| 1.10, 0.60, 1.31, 0.78, 0.52, 0.99, 1.14, 1.09 |
|2| 1.35, 1.32, 1.27, 0.64, 1.31, 0.56, 1.26, 0.87 |;
```

Please perform the following tasks

1. Create a ZIMPL file that for given (general)¹ I, J, R, S_0, S_1 checks whether the assets support type-A arbitrage.

¹General here means that instead of explicitly writing $|I|$ or $|J|$ many constraints, you should use the forall and sum syntax of ZIMPL

2. Create a ZIMPL file, that for given (general) I, J, R, S_0, S_1 computes risk neutral probabilities (if there are some).
3. Test both of your programs (i.e. translate it with the `zimpl` command and then solve the compiled LP with SOPLEX or QSOpt) to verify, whether the two examples above allow type-A arbitrage or whether risk neutral probabilities exist.
4. Send your two (compilable!) ZIMPL files to yanick.raja@epfl.ch **until the 30.10.09**. Together with this, send for each above example either a portfolio x that supports type-A arbitrage or a vector p of risk-neutral probabilities.

Hint: You can find a guide, how to install and use ZIMPL together with QSOpt (another LP-Solver) under Windows or MacOS by following

<http://disopt.epfl.ch/webdav/site/disopt/shared/OptInFinance08/zimplguideline.pdf>.