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Discussions from: December 03, 2013

Combinatorial Optimization

Fall 2013

Assignment Sheet 6

Exercise 1
For $A, B \subseteq U$ we have

$$\begin{aligned} g(A) + g(B) = f(A \cup H) + f(B \cup H) &\geq f(A \cup B \cup H) + f((A \cup H) \cap (B \cup H)) \\ &= g(A \cup B) + f((A \cap B) \cup H) = g(A \cup B) + g(A \cap B), \end{aligned}$$

where the inequality follows from the submodularity of f.

Exercise 2
See http://theory.stanford.edu/~jvondrak/CS369P-files/lec16.pdf, Sec. 3.

Exercise 3
See http://theory.stanford.edu/~jvondrak/data/submod-max-SICOMP.pdf, Lemma 2.3.

Exercise 4
See http://research.microsoft.com/en-us/um/people/roysch/Papers/SMC-BFNS14.pdf, Lemma 2.2

Exercise 5
See http://research.microsoft.com/en-us/um/people/roysch/Papers/SMC-BFNS14.pdf, Sec. 3.

Exercise 6
The reader should be easily able to deduce it from Theorem 39.13 and Corollary 39.12a from the book by Alexander Schrijver, *Combinatorial Optimization: Polyhedra and Efficiency*, Springer-Verlag.

Exercise 7 ($\star$)

a). Consider the set with 2 elements a and b, with $f(a) = 1$, and $f(S) = 0$ for $S = \emptyset, \{b\}, \{a, b\}$.

b). In the last passage of the proof seen in class for a generic nonnegative submodular function, we obtained:

$$f(ALG) \leq f(\emptyset) + f(OPT) + f(S \setminus OPT) + f(U),$$

where U is the universe set. For the cut function we have $f(OPT) = f(S \setminus OPT)$, and $f(ALG) \leq 2f(OPT)$ follows from nonnegativity.

As an alternative proof, consider the cut S obtained by taking each node with probability $1/2$. Each edge has probability $1/2$ of being in the cut, hence by linearity of expectation, the expected value of the number of edges in the cut is half of the total number of edges.