Exercises

Optimization in Finance

Fall 2008

Sheet 6

Exercise 6.1 - Strict Complementary Slackness
Suppose both systems

$$\begin{array}{ll} (P) & \min\{c^T x \mid Ax = b,\ x \geq \mathbf{0}\} \\ (D) & \max\{b^T y \mid y^T A \leq c^T\} \end{array}$$

are feasible. Then there are optimum solutions x^* for (P) and y^* for (D) such that for any i

$$x_i^* > 0 \Leftrightarrow y^{*T} a^i = c_i$$

with a^i being ith column of A.

Exercise 6.2
Proof the following theorem from the lecture: Let $K_1 < K_2 < \ldots < K_n$ denote strike prices of European call options on the same underlying security with same maturity. There are no arbitrage opportunities if and only if prices S_0^i satisfy

1. $S_0^i > 0$, $i = 1, \ldots, n$
2. $S_0^i > S_0^{i+1}$, $i = 1, \ldots, n-1$
3. $C(K_i) := S_0^i$ defined on $\{K_1, \ldots, K_n\}$ is a strictly convex function

Exercise 6.3
Consider a 2-person zero sum game with payoff matrix

$$A = \begin{pmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{pmatrix}$$

Compute the optimum mixed strategies for both players.

Exercise 6.4
Let $A = (a_{ij})_{1 \leq i \leq m, 1 \leq j \leq n} \in \mathbb{R}^{m \times n}$ be any matrix. Show that

$$\max_{i=1,\ldots,m} \min_{j=1,\ldots,n} a_{ij} \leq \min_{j=1,\ldots,n} \max_{i=1,\ldots,m} a_{ij}$$