Exercises

Optimization in Finance

Fall 2008

Sheet 6

Exercise 6.1 - Strict Complementary Slackness
Suppose both systems

\[(P) \quad \min \{ c^T x \mid Ax = b, \ x \geq 0 \} \]
\[(D) \quad \max \{ b^T y \mid y^T A \leq c^T \} \]

are feasible. Then there are optimum solutions \(x^*\) for \((P)\) and \(y^*\) for \((D)\) such that for any \(i\)

\[ x^*_i > 0 \iff y^* a_i = c_i \]

with \(a_i\) being \(i\)th column of \(A\).

Exercise 6.2
Proof the following theorem from the lecture: Let \(K_1 < K_2 < \ldots < K_n\) denote strike prices of European call options on the same underlying security with same maturity. There are no arbitrage opportunities if and only if prices \(S_0^i\) satisfy

1. \(S_0^i > 0, \ i = 1, \ldots, n\)
2. \(S_0^i > S_0^{i+1}, \ i = 1, \ldots, n - 1\)
3. \(C(K_i) := S_0^i\) defined on \(\{K_1, \ldots, K_n\}\) is a strictly convex function

Exercise 6.3
Consider a 2-person zero sum game with payoff matrix

\[ A = \begin{pmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{pmatrix} \]

Compute the optimum mixed strategies for both players.

Exercise 6.4
Let \(A = (a_{ij})_{1 \leq i \leq m, 1 \leq j \leq n} \in \mathbb{R}^{m \times n}\) be any matrix. Show that

\[ \max_{i=1,\ldots,m} \min_{j=1,\ldots,n} a_{ij} \leq \min_{j=1,\ldots,n} \max_{i=1,\ldots,m} a_{ij} \]