Prof. Friedrich Eisenbrand | Question session: 14.10.09
Location: MA A3 31 | Discussion: 21.10.09

Exercises

Optimization Methods in Finance

Fall 2009

Sheet 2

Exercise 2.1
We know that the dual of

$$(P) \quad \min\{c^T x \mid Ax = b;\ x \geq \mathbf{0}\}$$

is

$$(D) \quad \max\{b^T y \mid A^T y \leq c\}.$$

Use this fact to obtain the dual LP for

$$\min\{c^T x \mid Ax \geq b,\ x \geq \mathbf{0}\}.$$

Exercise 2.2
Consider the linear program

$$\begin{array}{rcl} \max\ x_1 - 2x_2 & & (P) \\ x_1 + x_2 & \leq & 4 \\ x_1 - 3x_2 & \leq & 6 \\ -2x_1 - x_2 & \leq & -3 \\ 4x_1 - 3x_2 & \leq & 15 \end{array}$$

1. State the dual program (to (P)).
2. The vector $x^* = (x_1^*, x_2^*) = (3, -1)$ is a unique optimal solution for (P) (you do not have to show that). Use this to obtain an optimum solution for the dual and argue why this solution is optimal for the dual.
   Hint: Use complementary slackness.

Exercise 2.3
Suppose we have the following European Call options, all w.r.t. the same underlying asset (and maturity) which is currently priced at 40 CHF:

Option i	strike price K_i (in CHF)	price S_0^i (in CHF)
1	30	10
2	40	7
3	50	10/3
4	60	0

Construct a portfolio of the above options that provides a type-A arbitrage opportunity.
Hint: You may use any LP solver.

Exercise 2.4
Suppose we are given 3 European Call options (all w.r.t. the same underlying asset, all with the same maturity), Option i with a price of S_0^i and strike price of K_i. Suppose that $K_1 < K_2 < K_3; S_0^1 > S_0^2 > S_0^3$ and the point (K_2, S_0^2) lies above (or on) the line segment that connects (K_1, S_0^1) and (K_3, S_0^3). Formally there is a $0 < \lambda < 1$ with $K_2 = \lambda K_1 + (1-\lambda)K_3$ and

$$S_0^2 \geq \lambda S_0^1 + (1-\lambda)S_0^3.$$

Give an *explicit* formula for a portfolio that provides arbitrage. Which type of arbitrage is it?