Exercises

Optimization Methods in Finance
Fall 2009
Sheet 1

Exercise 1.1
Write the following linear program in standard form

\[
\begin{align*}
\text{max } & 4x_1 + x_2 - x_3 \\
\text{s.t. } & x_1 + 3x_3 \leq 6 \\
& 3x_1 + x_2 + 3x_3 \geq 9 \\
& x_1, x_2 \geq 0 \\
& x_3 \in \mathbb{R}
\end{align*}
\]

Exercise 1.2
Draw the feasible region (set of feasible solutions) of the following linear program (with 2 variables)

\[
\begin{align*}
\text{max } & 2x_1 - x_2 \\
\text{s.t. } & x_1 + x_2 \geq 1 \\
& x_1 - x_2 \leq 0 \\
& 3x_1 + x_2 \leq 6 \\
& x_1, x_2 \geq 0
\end{align*}
\]

Determine the optimal solution to this problem by inspecting your drawing.

Exercise 1.3
i) For

\[
A = \begin{pmatrix}
1 & 0 & 1 \\
2 & 1 & 1 \\
3 & -1 & 4
\end{pmatrix}
\]

compute a \(d \in \mathbb{R}^3, d \neq 0\) with \(Ad = 0\).

ii) Invert the matrix

\[
B = \begin{pmatrix}
1 & 2 & 0 \\
2 & 3 & 0 \\
3 & 4 & 1
\end{pmatrix}
\]
iii) Compute a solution \( x \in \mathbb{R}^3 \) for \( Bx = b \) with \( b = (3, 1, -1)^T \).

**Exercise 1.4**

The vector \( x^* = (0, 1, 1, 1) \) is an optimal solution of

\[
\begin{align*}
\min (1,1,0,2) \cdot x \\
\begin{bmatrix}
0 & 1 & 2 & 3 \\
0 & 1 & 0 & 2 \\
2 & 0 & 0 & 0
\end{bmatrix} x &= \begin{bmatrix}
6 \\
3 \\
0
\end{bmatrix} \\
x &\geq 0
\end{align*}
\]

with \( x = (x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \). Use the proof of Lemma 2.1 to find another optimal solution \( x' \) such that \( A_J' \) has full column rank with \( J' = \{ i \mid x'_i > 0 \} \).

**Exercise 1.5 – Practical exercise (2 points)**

For the first practical exercise do the following (see the lecture notes for more details):

1. Transform the cashflow LP from the lecture into standard form.
2. Implement the naive linear programming algorithm to find an optimum solution (you can use the Boost C++-library).
3. Send the code together with compile instructions to thomas.rothvoss@epfl.ch until the 30th of September.