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Location: AAC 132 Discussion: 5.05.10

Exercises

Approximation Algorithms

Spring 2010

Sheet 9

Reminder: On May 5, lecture and tutorial are moved to AAC 132.

Exercise 1
Recall that for the k-TSP problem, we are given a complete graph $G = (V,E)$ with a metric cost function $c : E \to \mathbb{Q}_+$ and a parameter $k \in \{1,\dots,n\}$. The goal is to find a minimum length tour, visiting *at least k nodes*.

i) Show that if c is a tree metric (and you know the underlying tree T), then one can find an optimum tour in polynomial time.

ii) Give an expected $O(\log n)$-approximation algorithm for k-TSP in general metric graphs. Can you derandomize it?

Exercise 2
For STEINER FOREST, the input is a complete, undirected graph $G = (V,E)$ with metric cost function $c : E \to \mathbb{Q}_+$ and pairs $(s_1,t_1),\dots,(s_k,t_k)$ ($s_i,t_i \in V$). The goal is to find a min cost subgraph H, that connects each s_i-t_i pair:

$$OPT = \min_{H \subseteq E} \Big\{ \sum_{e \in H} c(e) \mid \forall i = 1,\dots,k : H \text{ connects } s_i \text{ and } t_i \Big\}$$

(there is no need to connect s_i,t_j for $i \neq j$, hence H itself does not need to be connected. In fact, in general it will be a *forest*, that is a collection of trees). Consider the following linear program

$$\begin{aligned} \min \sum_{e \in E} x_e c_e & & & \\ \sum_{e \in \delta(S)} x_e &\geq 1 & & \forall i = 1,\dots,k\ \forall S \subseteq V : s_i \in S, t_i \notin S \\ x_e &\geq 0 & & \end{aligned}$$

Here x_e can be interpreted as a variable that indicates whether e is included in H or not. Prove that the integrality gap of this LP is upperbounded by $O(\log n)$.