Exercises

Approximation Algorithms

Spring 2010

Sheet 9

Reminder: On May 5, lecture and tutorial are moved to AAC 132.

Exercise 1
Recall that for the \( k \)-TSP problem, we are given a complete graph \( G = (V,E) \) with a metric cost function \( c : E \to \mathbb{Q}_+ \) and a parameter \( k \in \{1, \ldots, n\} \). The goal is to find a minimum length tour, visiting at least \( k \) nodes.

i) Show that if \( c \) is a tree metric (and you know the underlying tree \( T \)), then one can find an optimum tour in polynomial time.

ii) Give an expected \( O(\log n) \)-approximation algorithm for \( k \)-TSP in general metric graphs. Can you derandomize it?

Exercise 2
For Steiner Forest, the input is a complete, undirected graph \( G = (V,E) \) with metric cost function \( c : E \to \mathbb{Q}_+ \) and pairs \((s_1,t_1), \ldots, (s_k,t_k) \) \( (s_i,t_i \in V) \). The goal is to find a min cost subgraph \( H \), that connects each \( s_i-t_i \) pair:

\[
\text{OPT} = \min_{H \subseteq E} \left\{ \sum_{e \in H} c(e) \mid \forall i = 1, \ldots, k : H \text{ connects } s_i \text{ and } t_i \right\}
\]

(there is no need to connect \( s_i, t_j \) for \( i \neq j \), hence \( H \) itself does not need to be connected. In fact, in general it will be a forest, that is a collection of trees). Consider the following linear program

\[
\begin{align*}
\min \sum_{e \in E} x_e c_e \\
\sum_{e \in \delta(S)} x_e &\geq 1 \quad \forall i = 1, \ldots, k \quad \forall S \subseteq V : s_i \in S, t_i \notin S \\
x_e &\geq 0
\end{align*}
\]

Here \( x_e \) can be interpreted as a variable that indicates whether \( e \) is included in \( H \) or not. Prove that the integrality gap of this LP is upperbounded by \( O(\log n) \).