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Exercises

Approximation Algorithms

Spring 2010

Sheet 7

Exercise 1
The following is called the SANTA CLAUS PROBLEM: Santa Claus has presents $1,\ldots,n$, that he wants to distribute among children $1,\ldots,m$, where p_{ij} is the value that kid i has for present j. Santa's goal is that the least luckiest kid is as happy as possible, that means he tries to achieve

$$OPT = \max_{I_1 \dot\cup \ldots \dot\cup I_m = \{1,\ldots,n\}} \left\{ \min_{i=1,\ldots,m} \left\{ \sum_{j \in I_i} p_{ij} \right\} \right\}$$

Suppose you know the value of OPT. Let $p_{\max} := \max_{i=1,\ldots,m} \max_{j=1,\ldots,n} p_{ij}$. Give a polynomial time algorithm that assigns the presents to children such that the happiness of every child is at least $OPT - p_{\max}$. Does this give you any approximation factor?
Hint: Probably you already noticed that this problem has much in common with the UNRELATED MACHINE SCHEDULING problem from the lecture.

Exercise 2
Recall that the following problem is **NP**-hard:

> 3-DIM MATCHING: Given disjoint sets $A = \{a_1,\ldots,a_n\}, B = \{b_1,\ldots,b_n\}, C = \{c_1,\ldots,c_n\}$ and tripels $F = \{T_1,\ldots,T_m\}$ ($|T_i| = 3$, $|T_i \cap A| = |T_i \cap B| = |T_i \cap C| = 1$). Decide, whether there is a *perfect 3-dim. matching*, i.e. a subset $F' \subseteq F$ of $|F'| = n$ disjoint tripels.

Let $t_j := |\{i \mid a_j \in T_i\}|$. We define an UNRELATED MACHINE SCHEDULING instance with machines $i = 1,\ldots,m$ and the following set of jobs

- For $j = 1,\ldots,n$ we have a job b_j with processing time

$$p_{i,b_j} = \begin{cases} 1 & \text{if } b_j \in T_i \\ \infty & \text{otherwise} \end{cases}$$

- For $j = 1,\ldots,n$ we have a job c_j with processing time

$$p_{i,c_j} = \begin{cases} 1 & \text{if } c_j \in T_i \\ \infty & \text{otherwise} \end{cases}$$

- For every $j = 1,\ldots,n$ we create jobs $D_{j,q}$ for $q = 1,\ldots,t_j - 1$ with

$$p_{i,D_{j,q}} = \begin{cases} 2 & \text{if } a_j \in T_i \\ \infty & \text{otherwise} \end{cases}$$

Perform the following tasks

i) Show that if there is a perfect 3-dim matching, then the optimum makespan is at most 2.

ii) Show that if there is no perfect 3-dim maching, then the makespan is at least 3.

iii) Which inapproximability factor do you obtain for UNRELATED MACHINE SCHEDULING?