Thomas Rothvoß
Location: ELD120 Discussion: 14.04.10

Exercises

Approximation Algorithms

Spring 2010

Sheet 6

Exercise 1

Let $I = (a_1, \dots, a_n)$ with $a_i \in [0,1]$ be a BIN PACKING instance. Consider the following algorithm

(1) Apply linear grouping with parameter k and call the emerging instance $I' = (a'_1, \dots, a'_k)$ (item a'_i appears $b_i \in \mathbb{N}_0$ times)

(2) Compute a near-optimal basic solution x of the Gilmore Gomory LP-relaxation for I'

(3) Buy $\lceil x_p \rceil$ times pattern $p \in \mathscr{P}$

Perform the following tasks:

i) Show that for a suitable choice of k, the above algorithm produces a solution that needs at most $OPT + O(\sqrt{n})$ bins.

ii) An *asymptotic FPTAS* for BIN PACKING is an algorithm that for any given $\varepsilon > 0$ finds a solution with at most $(1+\varepsilon)OPT + p(1/\varepsilon)$ bins where the running time must be polynomial in n and $1/\varepsilon$. Furthermore $p : \mathbb{R}_+ \to \mathbb{R}_+$ must be a polynomial. Show that if you run the above algorithm on the large items and distribute the small items afterwards (as usual), one obtains such an asymptotic FPTAS.
Hints: You will need a suitable threshold, to determine what a *small* item is.

Exercise 2

Again consider the BIN COVERING problem on instance $I = (a_1, \dots, a_n)$ ($a_i \in [0,1]$) with the restriction that $a_i \geq \delta$ for a universal constant $\delta > 0$. Adapt the algorithm from the previous exercise to obtain a solution covering $OPT - O(\sqrt{n})$ bins in polynomial time.
Hints: How would an adapted Gilmore-Gomory LP-relaxation look like? Show that under the assumption $a_i \geq \delta$ you can solve it optimally.