
Integer Points in Polyhedra

Spring 2009

Assignment Sheet 4

Exercise 1 (LLL-algorithm)

Prove that during the execution of the LLL-algorithm, all intermediate numbers have polynomial size.

Exercise 2 (Shortest vector)

Describe an efficient algorithm to find a shortest lattice vector in fixed dimension.

Exercise 3 (Closest vector)

Describe an efficient algorithm to find a closest lattice vector in fixed dimension.

Exercise 4 (Flatness theorem for ellipsoids)

Let E be an ellipsoid in \mathbb{R}^n . Show that if $E \cap \mathbb{Z}^n = \emptyset$, then there is an integral vector $c \in \mathbb{Z}^n$ such that

$$\max\{c^T x : x \in E\} - \min\{c^T x : x \in E\} \leq \omega(n),$$

where $\omega(n)$ is a constant depending only on dimension n .