Exercises

Approximation Algorithms

Spring 2010

Sheet 1

Exercise 1
Give family of undirected graphs $G = (V, E)$ and terminals $R$, such that asymptotically (i.e. for $|V| \to \infty$) the Minimum spanning tree is a factor 2 more expensive than the cheapest Steiner tree.

Exercise 2
For the Steiner Tree problem, we are given an undirected weighted graph $G = (V, E)$ and a set of terminals $R \subseteq V$. It is the goal to find a tree $T$ that connects all terminals. There exists a constant $c_0 > 1$ such that the following gap version of the 3-Set Cover problem is NP-hard:

Given sets $S_1, \ldots, S_m \subseteq \{1, \ldots, n\}$ with $|S_i| = 3$ and a parameter $k \in \mathbb{N}$, distinguish

- **Yes**: There is a cover with $\leq k$ sets
- **No**: There is no cover with $\leq c \cdot k$ sets

Show that Steiner Tree is APX-hard, i.e., show that there is a constant $c_1 > 1$ such that finding a $c_1$-approximate Steiner Tree is NP-hard.

**Hint**: Construct a Steiner Tree instance with 1 terminal for each element, 1 Steiner node per set and 1 special root terminal (unit cost edges should suffice).

Exercise 3
Consider the Maximum Coverage problem: Given sets $S_1, \ldots, S_m$ over a universe of elements $U = \{1, \ldots, n\} = \bigcup_{i=1}^{m} S_i$ and a parameter $k \in \mathbb{N}$. Choose $k$ sets that cover as many elements as possible, i.e.,

$$OPT := \max \left\{ |\bigcup_{i \in I} S_i| : |I| = k \right\}$$

Show that a straightforward greedy algorithm gives a $\frac{e}{e-1} \approx 1.58$-approximation.