Exercise 1
Show that $l^p_k = (\mathbb{R}^k, \|\cdot\|_p)$ is a norm for $p \geq 1$ and give an example that violates the triangle inequality for $p < 1$. Recall that for $x, y \in X = \mathbb{R}^k$, we defined

$$d_X(x, y) = \|x - y\|_p = \left(\sum_{i=1}^{k} |x_i - y_i|^p\right)^{\frac{1}{p}}$$

Exercise 2
Show that the following graph cannot be embedded into $l^k_2$ for any $k$.

![Graph Diagram]

Exercise 3
Let $f : X \to Y$ be an isometric embedding. Can $(X, d_X)$ be a true metric, and $(Y, d_Y)$ be a pseudometric? Can $(X, d_X)$ be a pseudometric, and $(Y, d_Y)$ be a true metric?

Exercise 4
If $(X, d_X)$ and $(Y, d_Y)$ are metric spaces. If $f : X \to Y$ is a bijective isometric embedding, is $f^{-1}$ a isometric embedding with the same metrics?

Exercise 5 [*]
Show that if $f : (X, d_X) \to (Y, d_Y)$ is bijective, then the distortion of $f$ equals $\|f\|_{Lip}\|f^{-1}\|_{Lip}$ where the distortion of $f$ is $\inf\{D : f$ is a $D-embedding\}$ and

$$\|f\|_{Lip} = \sup\left\{ \frac{d_Y(f(x), f(y))}{d_X(x, y)} : x, y \in X \right\}$$

Exercise 6
$C \subseteq \mathbb{R}^n$ is a convex cone if
- $\forall a \in C, \lambda \in \mathbb{R}_{\geq 0}, \lambda a \in C$
- $\forall a, b \in C, a + b \in C$

Show that a convex cone is a convex set.

The deadline for submitting solutions is Monday, November 9, 2015.