Convexity

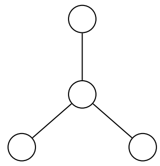
Prof. Friedrich Eisenbrand
Natalia Karaskova

Assignment Sheet 7
November 2, 2015

Exercise 1
Show that $l_p^k = (\mathbb{R}^k, \|\cdot\|_p)$ is a norm for $p \geq 1$ and give an example that violates the triangle inequality for $p < 1$. Recall that for $x, y \in X = \mathbb{R}^k$, we defined

$$d_X(x,y) = \|x-y\|_p = \left(\sum_{i=1}^{k} |x_i - y_i|^p\right)^{\frac{1}{p}}$$

Exercise 2
Show that the following graph cannot be embedded into l_2^k for any k.



Exercise 3
Let $f : X \to Y$ be an isometric embedding. Can (X, d_X) be a true metric, and (Y, d_Y) be a pseudometric? Can (X, d_X) be a pseudometric, and (Y, d_Y) be a true metric?

Exercise 4
If (X, d_X) and (Y, d_Y) are metric spaces. If $f : X \to Y$ is a bijective isometric embedding, is f^{-1} a isometric embedding with the same metrics?

Exercise 5 [$\star$]
Show that if $f : (X, d_X) \to (Y, d_Y)$ is bijective, then the distortion of f equals $\|f\|_{Lip}\|f^{-1}\|_{Lip}$ where the distortion of f is $\inf\{D : f \text{ is a D} - \text{embedding}\}$ and

$$\|f\|_{Lip} = \sup\left\{\frac{d_Y(f(x), f(y))}{d_X(x,y)} : x, y \in X\right\}$$

Exercise 6
$C \subseteq \mathbb{R}^n$ is a convex cone if
- $\forall a \in C, \lambda \in \mathbb{R}_{\geq 0}, \lambda a \in C$
- $\forall a, b \in C, a + b \in C$

Show that a convex cone is a convex set.

The deadline for submitting solutions is **Monday, November 9, 2015**.