Convexity

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Assignment Sheet 3

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Exercise 1
Let $D \subset \mathbb{R}^d$ be a disc of radius R, $D = \{x \in \mathbb{R}^d : ||x - r|| \leq R\}$. Show that, for $c \in \mathbb{R}^d$, we have

$$\max_{x \in D} c^\intercal x - \min_{x \in D} c^\intercal x = 2R||c||$$

Exercise 2
Draw a general lattice $\Lambda \subset \mathbb{R}^2$ and its dual lattice $\Lambda^\star$.

Exercise 3
Show that if $\Lambda \subset \mathbb{R}^d$ is a lattice, then there exist a non-zero point $x \in \Lambda$ with $||x|| \leq \sqrt{d}(det\Lambda)^{\frac{1}{d}}$

[*Hint:* You can use the fact that the volume of a unit ball $B \subset \mathbb{R}^d$ is $vol(B) = \frac{\pi^{\frac{d}{2}}}{\Gamma(\frac{d}{2}+1)}$]

Exercise 4 [$\star$]
Let $\Lambda \subset \mathbb{R}^d$ be a lattice and let $\Lambda^\star \subset \mathbb{R}^d$ be its dual lattice. Show that the packing radii of Λ and $\Lambda^\star$ satisfy

$$\rho(\Lambda)\rho(\Lambda^\star) \leq \frac{d}{4}$$

Exercise 5
For a lattice $\Lambda \subset \mathbb{R}^d$, we defined $\mu(\Lambda)$, the *covering radius* of Λ, as

$$\mu(\Lambda) = \max_{x \in \mathbb{R}^d} dist(x, \Lambda)$$

Show that $\mu(\lambda)$ is the minimal radius of a ball B such that $B + \Lambda \supseteq \mathbb{R}^d$.

The deadline for submitting solutions is **Monday, October 12, 2015**.