Convexity

Prof. Friedrich Eisenbrand
Natalia Karaskova

Assignment Sheet 11

December 7, 2015

Exercise 1 [$\star$]
Let $v_1, \ldots, v_n \in \mathbb{R}^n$, with $\|v_i\|_2 = 1$ for $i = 1, \ldots, n$. Using the probabilistic method, show that there exist $\epsilon_1, \ldots, \epsilon_n \in \{\pm 1\}$ such that

$$\|\epsilon_1 v_1 + \cdots + \epsilon_n v_n\|_2 \leq \sqrt{n}$$

and also that there exist $\epsilon_1, \ldots, \epsilon_n \in \{\pm 1\}$ such that

$$\|\epsilon_1 v_1 + \cdots + \epsilon_n v_n\|_2 \geq \sqrt{n}$$

Exercise 2
Let X be a finite set of N elements (assume that N is a power of 2). At least one of these elements is *interesting*. You may ask questions of the form: *Does $H \subseteq X$ contain an interesting element?* Design an algorithm that identifies an interesting element by asking at most $\log_2(N)$ questions of this kind.

Can there exist a deterministic algorithm that identifies an interesting element by asking fewer questions in the worst case?

Exercise 3
Show the following inequalities for $0 \leq \epsilon \leq \frac{1}{2}$:

1. $(1 - \epsilon)^x \leq (1 - \epsilon x)$ for $x \in [0, 1]$
2. $(1 + \epsilon)^{-x} \leq (1 - \epsilon x)$ for $x \in [-1, 0]$
3. $\ln\left(\frac{1}{1-\epsilon}\right) \leq \epsilon + \epsilon^2$
4. $\ln(1 + \epsilon) \geq \epsilon - \epsilon^2$

The deadline for submitting solutions is **Monday, December 14, 2015**.