Problem 1
Let $C$ be the square with corners at $(1,1), (1,-1), (-1,1), (-1,-1)$. Draw the polar of $C$. Now consider the square $C'$ with corners at $(1,1), (1,0), (0,1), (0,0)$ and draw its polar. Why is it different from the previous one?

Problem 2
Let $P \subseteq \mathbb{R}^n$ be a full dimensional polytope that contains the origin $0$ in its interior. Let $x \in \mathbb{R}^n$. Prove that $x$ is a vertex of $P$ if and only if $\{y \in \mathbb{R}^n | x^\top y \leq 1\}$ defines a facet of $P^0$. (Hint: use the fact that any point in $P$ can be expressed as a convex combination of the vertices in $P$).

Problem 3
Let $M(E, I)$ be a matroid with rank function $r$, $x^* \in \mathbb{R}^n_+$, $f : 2^E \to \mathbb{R}$ defined as $f(A) = r(A) - x^*(A)$. Prove that $f$ is submodular. Deduce that one can solve the separation problem for the matroid polytope by solving the problem of minimizing a submodular function.

Problem 4
We are given a graph $G(V, E)$ and a coloring of its edges. We want to find whether there exists a spanning tree $T$ of $G$ such that no two edges of $T$ have the same color. Show how to solve this problem in polynomial time.