École Polytechnique Fédérale de Lausanne
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Combinatorial Optimization (Fall 2016)

Assignment 12

Problem 1

Let C be the square with corners at $(1,1),(1,-1),(-1,1),(-1,-1)$. Draw the polar of C. Now consider the square C' with corners at $(1,1),(1,0),(0,1),(0,0)$ and draw its polar. Why is it different from the previous one?

Problem 2

Let $P \subseteq \mathbb{R}^n$ be a full dimensional polytope that contains the origin $\mathbf{0}$ in its interior. Let $x \in \mathbb{R}^n$. Prove that x is a vertex of P if and only if $\{y \in \mathbb{R}^n | x^\top y \leq 1\}$ defines a facet of P^0. (Hint: use the fact that any point in P can be expressed as a convex combination of the vertices in P).

Problem 3

Let $M(E, \mathcal{I})$ be a matroid with rank function r, $x^* \in R^n_+$, $f : 2^E \to \mathbb{R}$ defined as $f(A) = r(A) - x^*(A)$. Prove that f is submodular. Deduce that one can solve the separation problem for the matroid polytope by solving the problem of minimizing a submodular function.

Problem 4

We are given a graph $G(V, E)$ and a coloring of its edges. We want to find whether there exists a spanning tree T of G such that no two edges of T have the same color. Show how to solve this problem in polynomial time.