Prof. F. Eisenbrand EPFL - DISOPT

Combinatorial Optimization

Adrian Bock Fall 2011

Sheet 1 **September 22, 2011**

General remark:
In order to obtain a bonus for the final grading, you may hand in written solutions to the exercise marked with a star at the beginning of the exercise session on October 4.

Exercise 1
Let $A \in \mathbb{R}^{m \times n}, b \in \mathbb{R}^m$ and $c \in \mathbb{R}^n$. Show that the dual of the linear program

$$\max\{c^T x \colon x \in \mathbb{R}^n,\, Ax \le b,\, x \ge 0\}$$

can be interpreted as the linear program

$$\min\{b^T y \colon y \in \mathbb{R}^m,\, A^T y \ge c,\, y \ge 0\}.$$

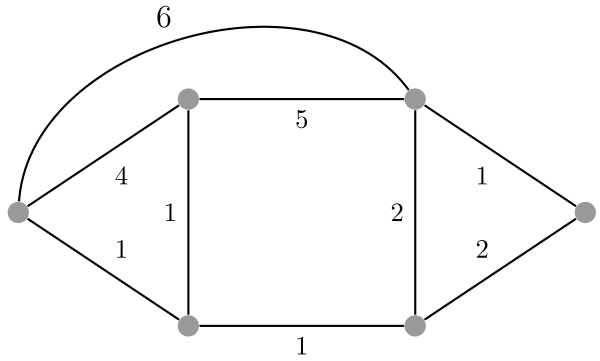
In addition, mark (and argue!) the entries of the following table corresponding to possible outcomes in this primal/dual pair of linear programs:

		Dual		
		Finite optimum	Unbounded	Infeasible
Primal	Finite optimum			
	Unbounded			
	Infeasible			

Exercise 2
Determine a maximum weight matching of the graph below. Provide of proof of optimality by determining a feasible dual solution to the linear program

$$\begin{array}{rrcl}
\max & \sum_{e \in E} w(e)x(e) & & \\
v \in V: & \sum_{e \in \delta(v)} x(e) & \le & 1 \\
\substack{U \subseteq V \\ |U| \text{ odd}}: & \sum_{e \in E(U)} x(e) & \le & \lfloor |U|/2 \rfloor \\
e \in E: & x(e) & \ge & 0
\end{array}$$

whose objective value coincides with the weight of your matching.



Exercise 3 ($\star$)

In the following, we consider undirected graphs without edge weights.

Suppose that we have an efficient algorithm to decide whether a graph contains a perfect matching (i.e. each node is incident with exactly one edge).
Use that algorithm to determine efficiently the size of a maximum matching in a given graph $G = (V, E)$.

Exercise 4

Show the following: A face F of $P = \{x \in \mathbb{R}^n : Ax \leq b\}$ is inclusion-wise minimal if and only if it is of the form $F = \{x \in \mathbb{R}^n \mid A'x = b'\}$ for some subsystem $A'x \leq b'$ of $Ax \leq b$.