École Polytechnique Fédérale de Lausanne
Prof. Eisenbrand November 25, 2016
Assistant: Manuel Aprile

Combinatorial Optimization (Fall 2016)

Assignment 8

Deadline: December 2 10:00, into the right box in front of MA C1 563.

Exercises marked with a $\star$ can be handed in for bonus points.

Problem 1

We saw that the description of the matching polytope of a graph $G(V, E)$ contains an inequality for each $U \subseteq V$ such that $|U|$ is odd. If $|V| = n$, how many such inequalities are there? Is there any of those inequalities that is always redundant? (An inequality is redundant if it can be removed from the description without changing the polytope).

Problem 2

Prove that G has a perfect matching if and only if for any $W \subseteq V$, the graph $G \setminus W$ obtained by removing W has at most $|W|$ odd components. To prove the "if" direction, use the fact that the system of inequalities describing the matching polytope is Totally Dual Integral.

Problem 3 ($\star$)

Prove the following lemma used in class: let $G(V, E)$ be a connected graph and $w : E \to \mathbb{N}_{\geq 1}$, at least one of the following must hold:

a) There is a vertex $v \in V$ such that $\delta(v) \cap M \neq \emptyset$ for any $M \in \mathcal{M}(w)$.

b) $z(w') = z(w) - \lfloor \frac{|V|}{2} \rfloor$, and $|V|$ is odd.

Where: $w' = w - \vec{1}$ is the vector w with each entry decreased by 1; $\mathcal{M}(w)$ is the set of maximum weight matchings with respect to w (similarly for w'); $z(w) = w(M)$ for any $M \in \mathcal{M}(w)$ (similarly for w').