Problem 1
Determine the dual program for the following linear program:
\[
\begin{align*}
\text{min} & \quad 3x_1 + 2x_2 - 3x_3 + 4x_4 \\
& \quad 2x_1 - 2x_2 + 3x_3 + 4x_4 \leq 3 \\
& \quad x_2 + 3x_3 + 4x_4 \geq -5 \\
& \quad 2x_1 - 3x_2 - 7x_3 - 4x_4 = 2 \\
& \quad x_1 \geq 0 \\
& \quad x_4 \leq 0
\end{align*}
\]

Problem 2
In the GitHub repository of the course, in the ‘Programming’ folder, you will find the file ‘Simplex.py’ which contains the code to run the simplex algorithm on non-degenerate linear programs in standard form. However, some code is missing. Complete the code by filling the gaps (indicated by underscores) and send it to igor@malinovic.epfl.ch.

Problem 3 (⋆)
Suppose you are given an oracle algorithm, which for a given polyhedron
\[
P = \{ \tilde{x} \in \mathbb{R}^n : \tilde{A}\tilde{x} \leq \tilde{b} \}
\]
gives you a feasible solution or asserts that there is none.
Consider the LP \(\max \{ c^T x : Ax \leq b, \ x \in \mathbb{R}^n \} \), and assume it is feasible and bounded. Show that one can obtain an optimal solution of the LP using a single call of the oracle algorithm on a suitable polyhedron. \textit{Hint: Use duality theory!}

Problem 4
Consider the following linear program:
\[
\begin{align*}
\text{max} & \quad x_1 + x_2 \\
\text{subject to} & \quad 2x_1 + x_2 \leq 6 \\
& \quad x_1 + 2x_2 \leq 8 \\
& \quad 3x_1 + 4x_2 \leq 22 \\
& \quad x_1 + 5x_2 \leq 23
\end{align*}
\]
Show that \((4/3, 10/3)\) is an optimal solution by using weak duality.

Problem 5
Let \(P = \{ x \in \mathbb{R}^n : Ax \leq b \}\) a bounded, non-empty polyhedron. Formulate a linear program that computes the largest ball inside \(P\).