École Polytechnique Fédérale de Lausanne
Prof. Eisenbrand
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November 11, 2016

Combinatorial Optimization (Fall 2016)

Assignment 6

Deadline: November 18 10:00, into the right box in front of MA C1 563.

Exercises marked with a $\star$ can be handed in for bonus points.

Problem 1

Let $M = (X, \mathcal{I})$ be a matroid, prove the following:

1. Every basis B of M (i.e. maximal independent set) has the same cardinality.
2. (Basis exchange property) Given bases B, B' of M, for any $x \in B \setminus B'$ there is a $y \in B' \setminus B$ such that $B \setminus \{x\} \cup \{y\}$ is a basis.

Problem 2 ($\star$)

Let $M = (X, \mathcal{I})$ be a matroid, with $X = \{x_1, \ldots, x_m\}$. Prove that the set

$$Y = \{x_i \text{ such that } rk(x_1, \ldots x_i) > rk(x_1, \ldots x_{i-1})\}$$

is independent (i.e. $Y \in \mathcal{I}$).

Problem 3 ($\star$)

Let $P = \{x \in \mathbb{R}^n : Ax \leq b\}$.

1. Prove that for any vertex v of P, there is a direction $w \in \mathbb{R}^n$ such that v is the unique optimal solution of the LP $\max\{wx : x \in P\}$.
2. Assume that, for any $w \in \mathbb{R}^n$, the LP $\max\{wx : x \in P\}$ is either unbounded or admits as optimal solution an integral vertex. Prove that P is integral (i.e., that all vertices of P have integer coordinates).