École Polytechnique Fédérale de Lausanne
Prof. Eisenbrand
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October 21, 2016

Combinatorial Optimization (Fall 2016)

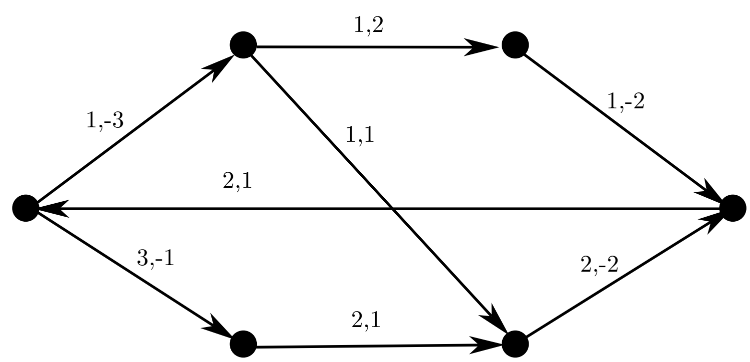
Assignment 4

Deadline: October 28 10:00, into the right box in front of MA C1 563.

Exercises marked with a $\star$ can be handed in for bonus points.

Problem 1

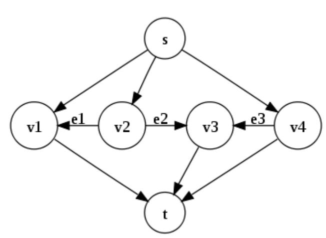
Determine in the following networks a circulation of minimum cost using the negative cycle canceling algorithm (the numbers on the arcs stand for (capacity, cost)):



Problem 2 ($\star$)

1. Prove that, in a directed graph with integral capacities, the negative cycle canceling algorithm always terminates.

2. Consider the following graph:



   where the capacities of e_1, e_2, e_3 are 1, 1, $r = \frac{\sqrt{5}-1}{2}$ (note that r satisfies $r^2 = 1 - r$), and the other edges have as capacity some integer $M \geq 2$. Show that the Ford Fulkerson algorithm does not terminate on this graph if the wrong paths are chosen. You should start with the path sv_2v_3t and then choose the following paths in the right order: $P_A = sv_1v_2v_3t$, $P_B = sv_4v_3v_2v_1t$, $P_C = sv_2v_3v_4t$, so that flow can be pushed through the sequence of paths infinitely many times. Does the flow given by this procedure converge to the maximum flow (which is $2M + 1$)?

3. Give an example of a graph with real capacities such that negative cycle canceling algorithm does not terminate. (Hint: transform the example from above in a minimum cost circulation instance, adding extra arcs and giving costs).

Problem 3

Given a directed graph D with capacities c and costs w on the arcs, a feasible $s-t$ flow f of D is *extreme* if for any $s-t$ flow f' with the same value as f, $w(f') \geq w(f)$. For some value $M \in \mathbb{R}_+$, we want to know if there exist an extreme $s-t$ flow of value M. Model this problem as a minimum cost circulation problem.