Exercises marked with a ⋆ can be handed in for bonus points.

**Problem 1**
Determine in the following networks a circulation of minimum cost using the negative cycle canceling algorithm (the numbers on the arcs stand for (capacity, cost)):

![Network Diagram]

**Problem 2 (⋆)**

1. Prove that, in a directed graph with integral capacities, the negative cycle canceling algorithm always terminates.

2. Consider the following graph:

![Graph Diagram]

where the capacities of $e_1, e_2, e_3$ are 1, 1, $r = \frac{\sqrt{5} - 1}{2}$ (note that $r$ satisfies $r^2 = 1 - r$), and the other edges have as capacity some integer $M \geq 2$. Show that the Ford Fulkerson algorithm does not terminate on this graph if the wrong paths are chosen. You should start with the path $sv_2v_3t$ and then choose the following paths in the right order: $P_A = sv_1v_2v_3t$, $P_B = sv_4v_3v_2v_1t$, $P_C = sv_2v_3v_4t$, so that flow can be pushed through the sequence of paths infinitely many times. Does the flow given by this procedure converge to the maximum flow (which is $2M + 1$)?

3. Give an example of a graph with real capacities such that negative cycle canceling algorithm does not terminate. (Hint: transform the example from above in a minimum cost circulation instance, adding extra arcs and giving costs).
Problem 3
Given a directed graph $D$ with capacities $c$ and costs $w$ on the arcs, a feasible $s-t$ flow $f$ of $D$ is extreme if for any $s-t$ flow $f'$ with the same value as $f$, $w(f') \geq w(f)$. For some value $M \in \mathbb{R}_+$, we want to know if there exist an extreme $s-t$ flow of value $M$. Model this problem as a minimum cost circulation problem.