École Polytechnique Fédérale de Lausanne
Prof. Eisenbrand

March 8, 2018

Discrete Optimization (Spring 2018)

Assignment 3

Problem 6 can be **submitted** until March 16 12:00 noon into the box in front of MA C1 563. You are allowed to submit your solutions in groups of at most three students.

Problem 1

Show that the recursion $T(n) = 8 \cdot T(n/2) + \theta(n^2)$, with the initial condition $T(1) = \theta(1)$, has the solution $T(n) = \theta(n^3)$.

Problem 2

Let I be an index set and $C_i \subseteq \mathbb{R}^n$ be a convex set for each $i \in I$, prove that $\cap_{i \in I} C_i$ is a convex set (Proposition 3.1 in the lecture notes).

Problem 3

Given an extreme point v of a convex set K. Show that v cannot be written as a convex combination of other points in K.

Problem 4

Let (1) be a linear program in inequality standard form, i.e.

$$\max\{c^{\mathrm{T}}x \mid Ax \leq b, x \in \mathbb{R}^n\} \tag{1}$$

where $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$, and $c \in \mathbb{R}^n$.
Prove that there is an equivalent linear program (2) of the form

$$\min\{\tilde{c}^{\mathrm{T}}y \mid \tilde{A}y = \tilde{b}, y \geq 0, y \in \mathbb{R}^{\tilde{n}}\} \tag{2}$$

where $\tilde{A} \in \mathbb{R}^{\tilde{m} \times \tilde{n}}$, $\tilde{b} \in \mathbb{R}^{\tilde{m}}$, and $\tilde{c} \in \mathbb{R}^{\tilde{n}}$ are such that every optimal point of (1) corresponds to an optimal point of (2) and vice versa.
Linear programs of the form (2) are said to be in *equality standard form.*

Problem 5

Consider the following linear program:

$$\begin{aligned} \max \quad & a + 3b \\ \text{s.t.} \quad & a + b \leq 2 \\ & a \leq 1 \\ & -a \leq 0 \\ & -b \leq 0 \end{aligned}$$

Compute the optimal solution via vertex enumeration. Give an alternative proof/certificate that the vertex you found is an optimal solution.

Problem 6 ($\star$)

Suppose that $A \in \mathbb{R}^{m \times n}$ has full-column rank and x^* is a feasible solution to $Ax \leq b$. Provide an algorithm that computes an extreme point of $P = \{x : Ax \leq b\}$ in polynomial time in the dimension and the encoding length of A, b, x^*.
Hint: See the proof of Theorem 3.2 in the lecture notes.