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Combinatorial Optimization

Fall 2010

Assignment Sheet 3

Exercise 1

1. Show that the unit simplex $\Delta = \text{conv}\{0, e_1, \dots, e_n\} \subset \mathbb{R}^n$, where e_j are the standard unit vectors, has volume $\frac{1}{n!}$.
2. Show that the volume of the simplex $\text{conv}\{v_0, v_1, \dots, v_n\} \subset \mathbb{R}^n$ is $\frac{|\det(v_1 - v_0, \dots, v_n - v_0)|}{n!}$.

Exercise 2 ($\star$)
Describe an algorithm that given as input

- an integral polyhedron $P = \{x \in \mathbb{R}^n \mid Ax \le b\}$, where $A \in \mathbb{R}^{m \times n}$ and $b \in \mathbb{R}^m$,
- an objective function vector $c \in \mathbb{Z}^n$,
- the optimal objective function value $z = \max\{c^T x \mid x \in P\}$, and
- a feasible point $x^\star \in P$ with $z - \frac{1}{2} \le c^T x^\star \le z$

computes an inclusion-wise minimal optimal face $F = \{x \in \mathbb{R}^n \mid A'x = b'\}$ of P. The running time of your algorithm shall be bounded by a polynomial in n and m.

Exercise 3
Reduce the problem of finding a maximum weight matching to the problem of finding a maximum weight *perfect* matching. That is, find a way to transform, in polynomial time, any graph G with edge weights w into a graph G' with edge weights w' so that given a maximum weight perfect matching M' of G', one can easily deduce a maximum weight matching M of G.

Exercise 4 ($\star$)
Let $\mathscr{F} \subseteq \{0,1\}^n$ and assume access to an oracle that given $\bar{x} \in \mathscr{F}$ and $\bar{c} \in \mathbb{Z}^n$ either

- asserts that $\bar{x} \in \mathscr{F}$ maximizes $\bar{c}^T x$ over $x \in \mathscr{F}$, or
- returns an $x \in \mathscr{F}$ such that $\bar{c}^T x > \bar{c}^T \bar{x}$.

Describe an algorithm that given $c \in \mathbb{Z}^n$ and an initial feasible solution $x \in \mathscr{F}$ computes an optimal solution of $\max\{c^T x \mid x \in \mathscr{F}\}$ with a running time that is bounded by a polynomial in n and $\log |c|_\infty$.