

Combinatorial Optimization

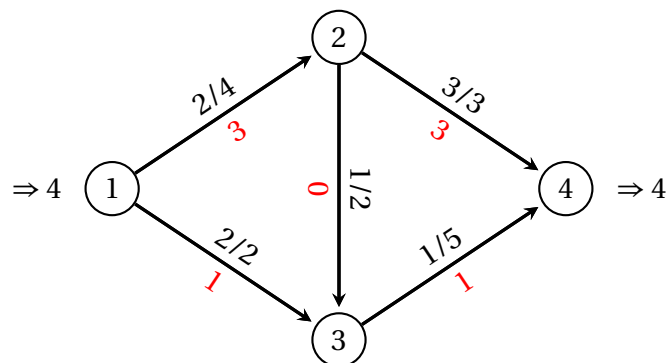
Fall 2015

Assignment Sheet 3

Exercises marked with a ★ can be handed in for bonus points. Due date is Friday October 9.

Exercise 1

In this exercise we are going to finish the example we started in class for the minimum cost network flow problem. The graph below shows our instance together with an initial feasible flow of value 4 depicted in red. Run the negative cycle canceling algorithm on this instance until you find an optimal flow. At each step clearly depict the residual network and the negative cycle used for augmentation.



Exercise 2

In each iteration of the negative cycle-canceling algorithm we have to find a negative cost cycle in the residual network or determine that such a cycle does not exist. Show how this can be done efficiently. **Hint:** use the Bellman-Ford algorithm.

Exercise 3

Let f be a feasible flow for an instance of MCNFP and $D(f) = (V, A_f)$ the corresponding residual network. In class we have shown that f is optimal if and only if there are not negative cost cycles in $D(f)$. In this exercise we will develop another optimality condition.

Assume that we associate with each node $v \in V$ a potential $\pi(v)$. We then define the reduced costs c^π for each arc $e = (u, v) \in A_f$ as

$$c^\pi(e) = c(e) + \pi(v) - \pi(u)$$

where $c(e)$ is the cost of arc e in the residual network $D(f)$.

a) Show that for any cycle C of $D(f)$ we have

$$\sum_{e \in C} c(e) = \sum_{e \in C} c^\pi(e)$$

b) Show that for any directed (u, v) -path P in $D(f)$ we have

$$\sum_{e \in P} c^\pi(e) = \sum_{e \in P} c(e) + \pi(v) - \pi(u)$$

c) [★] Show that the feasible flow f is optimal if and only if there exists a potential function π such that $c^\pi(e) \geq 0$ for all $e \in A_f$.

d) Find such a potential function π for the optimal flow you found in Exercise 1.

Exercise 4

Provide an example of MCNFP with integral capacities and costs for which the number of iterations of the negative cycle canceling algorithm can be exponential in the binary encoding lengths of the input if the negative cycles are chosen in a disadvantageous way.