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Discussions from: September 23, 2015

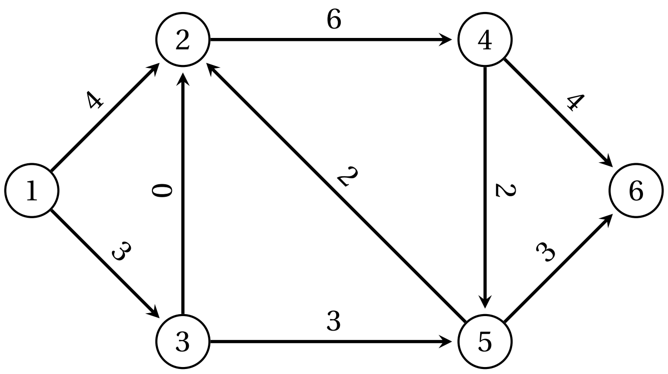
Combinatorial Optimization

Fall 2015

Assignment Sheet 2

Exercises marked with a ⋆ can be handed in for bonus points. Due date is Friday October 2.

Exercise 1
Consider the arc flow f on the network below. Decompose f as a path and cycle flow using the flow decomposition algorithm seen in class. Show all your steps.



Exercise 2
Consider the minimum cost network flow problem (MCNFP) discussed in class. Let $D = (V, A)$ be a directed graph with capacities $u : A \to \mathbb{R}_{\geq 0}$, costs $c : A \to \mathbb{R}$, and external flow $b : V \to \mathbb{R}$. Show how to transform the MCNFP problem for (D, u, c, b) to a MCNFP problem on a directed graph $D' = (V', A')$ with capacities $u' : A' \to \mathbb{R}_{\geq 0}$, costs $c' : A' \to \mathbb{R}$, and external flow $b' : V' \to \mathbb{R}$ such that $b'(i) = 0$ for all $i \in V'$. Argue that you can transform an optimal solution for MCNFP on (D', u', c', b') to an optimal solution for MCNFP on (D, u, c, b).

Hint: create two additional nodes: a source s and a sink t. Include an arc (t, s) as well as an arc (s, i) for each $i \in V$ with $b(i) > 0$ and an arc (i, t) for each $i \in V$ with $b(i) < 0$. Define appropriate costs and capacities for the new arcs.

Exercise 3 (⋆)
Let $D = (V, A)$ be a directed graph with capacities $u : A \to \mathbb{R}_{\geq 0}$, costs $c : A \to \mathbb{R}$ (some of which may be negative) and external flow $b : V \to \mathbb{R}$. Show how to transform the MCNFP problem for (D, u, c, b) to a MCNFP problem on a directed graph $D' = (V', A')$ with capacities $u' : A' \to \mathbb{R}_{\geq 0}$, costs $c' : A' \to \mathbb{R}_{\geq 0}$ and external flow $b' : V' \to \mathbb{R}$. Explain how to transform an optimal solution for MCNFP on (D', u', c', b') to an optimal solution for MCNFP on (D, u, c, b). You may assume that the original capacities u are all finite.