

## Combinatorial Optimization

Fall 2015

### Assignment Sheet 11

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★ exercises can be handed in for bonus points. Due date is Friday December 4.

#### Exercise 1

Determine whether the following statements are true or false and provide justification.

- let  $X$  be a decision problem in NP. If  $SAT$  is in  $P$  then  $X$  must also be in  $P$ .
- let  $X$  be a decision problem in NP. If  $P = NP$  then  $X$  is NP-complete.
- let  $X_1$  and  $X_2$  be decision problems in NP and assume that  $P \neq NP$ . If  $X_1$  is polynomial time reducible to  $X_2$  and  $X_2$  is polynomial time reducible to  $X_1$  then both  $X_1$  and  $X_2$  are NP-complete.

#### Exercise 2

In class we have seen the Hamiltonian cycle problem (HCP) which is to determine whether for a given graph  $G$  there exists a cycle that contains every vertex of  $G$ . The Hamiltonian path problem (HPP) on the other hand is to determine whether there is a simple path that contains every vertex of  $G$ . Show that there is a polynomial time reduction of HCP to HPP.

#### Exercise 3 (★)

Integer programming (IP) is the following decision problem: given a matrix  $A \in \mathbb{Q}^{m \times n}$  and a vector  $b \in \mathbb{Q}^m$  decide whether the set

$$\{x \in \mathbb{Z}^n \mid Ax \leq b\}$$

is non-empty. Show that IP is NP-complete.