Prof. Friedrich Eisenbrand
Gennady Shmonin

Due Date: November 26, 2008

Combinatorial Optimization

Fall 2008

Assignment Sheet 5

Exercise 1 (Optimum vertex solution)
Suppose that a linear program $\max\{c^T x : Ax \leq b\}$ can be solved in polynomial time. Yet, suppose that $P(A,b)$ has vertices and that the linear program is bounded. Show how to compute an optimal vertex solution of the linear program in polynomial time.

Exercise 2 (Totally unimodular matrices)
Let A be a totally unimodular matrix. Prove the following statements:

(a) A is totally unimodular if and only if A^T is totally unimodular.

(b) A is totally unimodular if and only if $[A\ I]$ is unimodular.

(c) A is totally unimodular if and only if the matrix $\begin{bmatrix} A \\ -A \\ I \\ -I \end{bmatrix}$ is totally unimodular.

(d) A is totally unimodular if and only if each nonsingular submatrix of A has a row with an odd number of nonzero components.

(e) A is totally unimodular if and only if the sum of the entries in any square sub- matrix with even row and column sums is divisible by four.

Exercise 3 (Kőnig's theorem)
Let $G = (V,E)$ be a graph. A subset of vertices $U \subseteq V$ is called a *vertex cover* if for each edge $e \in E$, $U \cap e \neq \emptyset$. Prove that if G is bipartite, then the maximum size of a matching is equal to the minimum size of a vertex cover in G.

Exercise 4 (Bipartite matchings)
Present a polynomial-time algorithm for computing a maximum-size matching in a bipartite graph.