Randomized Algorithms. Exercises for 17.11

pr.5.5 Use probabilistic method to show existence of an expanding bipartite graph $(L, R, E)$ with the following properties:

- $|L| = |R| = n$.
- Every vertex in $L$ has degree $\frac{3n}{4}$, and every vertex in $R$ has degree at most $3\frac{n}{4}$.
- Every subset of $\frac{n}{4}$ vertices is $L$ has at least $n - \frac{n}{4}$ neighbors in $R$.

pr.7.16 Use definition of $IP$ to directly get:

- $NP \subseteq IP$,
- If definition of $IP$ is modified to require probability of error being zero, then the resulting class in $NP$,
- $co-RP \subseteq IP$.

pr.7.16 Show that $IP \subseteq PSPACE$

pr.7.18 Define $MIP$ as an extension of $IP$, where the verifier has access to two provers who cannot communicate with each other. Show that $MIP = PCP$.

pr.7.19 Show that:

- $P = PCP[0, 0]$,
- $NP = PCP[0, poly(n)]$,
- $co-RP = PCP[poly(n), 0]$. 