Convexity

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Assignment Sheet 11

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Exercise 1
For $\lambda > 0$, show $\sum_{i=1}^{\infty} \frac{\lambda^{2i}}{(2i)!} \leq e^{\lambda^2/2}$ which we used to show the Chernoff bound.

Exercise 2
Let $N(\mu, \sigma^2)$ denote the probability distribution $\Pr[X] = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{(X-\mu)^2}{\sigma^2}}$. Prove the following Lemma.

Let $X_1, \dots, X_n$ be independent random variables with distribution $N(0, 1)$ and let $a = (a_1, \dots, a_n)^\intercal \in \mathbb{R}^n$. Then $\sum_{i=1}^{n} a_i X_i$ has distribution $N(0, \|a\|_2^2)$.

[*Hint: Start with only two independent random variables X and Y. How does the density function for (X, Y) look like? What happens if you apply a rotation to (X, Y)? Can you use this to show $c_1 X + c_2 Y \sim N(0, 1)$ for constants c_1, c_2 with $c_1^2 + c_2^2 = 1$? For $a > 0$, how does the density function for aX look like? Can you go on from there?*]

Exercise 3
Let $X \sim N(0, 1)$. Find a constant $a > 0$ such that $\Pr(X > \lambda) \leq e^{-a\lambda^2}$ for all $\lambda > 0$.

Exercise 4
Show the other estimation for the Gaussian Annulus theorem. This is, show the following.
For $X \sim N(0, 1), X \in \mathbb{R}^n$, show that $\Pr[\|X\| \leq \sqrt{n} - \beta] \leq e^{-\frac{c\beta^2}{2}}$, where $\beta > 0$ and c is a global constant.

The deadline for submitting solutions is **Wednesday, December 21, 2016 at 12 o'clock**.