Convexity

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Assignment Sheet 6

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Exercise 1
Let P be a polyhedron and F a facet. Show $\dim(F) = \dim(P) - 1$.

Exercise 2
Prove the Birkhoff- von Neumann Theorem:
The extreme points of the set of doubly stochastic matrices

$$M = \left\{ A = (a_{i,j})_{1 \le i,j \le n} \in \mathbb{R}^{n \times n}_{\ge 0} \mid \sum_{i=1}^{n} a_{i,j_0} = \sum_{j=1}^{n} a_{i_0,j} = 1 \ \forall 1 \le i_0, j_0 \le n \right\}$$

are precisely the permutation matrices, i.e. $M \cap \{0,1\}^{n \times n}$.

Exercise 3 [$\star$]
Using the separation theorem, prove that the system $Ax = b$, $x \ge 0$ has no solution if and only if there is a vector c s.th. $c^T A \ge 0$ and $c^T b < 0$.

Exercise 4
Let F be an (inclusion-wise) minimal face of a polyhedron $P = \{x : \ Ax \le b\}$. Show that the following holds.

$$\forall x, y \in F : \ Ax = Ay.$$

The deadline for submitting solutions is **Thursday, November 10, 2016**.