Assignment Sheet 9
December 1, 2016

Exercise 1
The Gamma function if defined as \( \Gamma(x) = \int_0^\infty r^{x-1}e^{-r}dr \) for \( x > 0 \). Prove that

1. \( \Gamma(x + 1) = x\Gamma(x) \).
2. \( \Gamma(n) = (n - 1)! \) for positive integer \( n \).

Exercise 2
Prove the missing step in the computation of \( v_n \), the volume of the ball \( B^1_n \), from the lecture. Namely show that

\[
\int_0^{\infty} nR^{n-1}e^{-R^2/2}dR = 2^n \Gamma\left( \frac{n}{2} + 1 \right)
\]

Exercise 3 [⋆]
Let \( A, B \subseteq \mathbb{R}^n \) be convex sets. Show the following equality of sets (in \( \mathbb{R}^{n+1} \)).

\[
\text{conv} \left( ([0] \times A) \cup ([1] \times B) \right) = \bigcup_{t \in [0,1]} \left[ [t] \times ((1-t)A + tB) \right].
\]

Exercise 4
Let \( A \subseteq \mathbb{R}^n \) be a brick set consisting of at least two bricks. Show that there exist a canonic unit vector \( e_i \in \mathbb{R}^n \) and \( b \in \mathbb{R} \) and two bricks \( B_1, B_2 \in A \) s.t. \( e_i^T x \leq b \) for all \( x \in B_1 \) and \( e_i^T x \geq b \) for all \( x \in B_2 \). That is, show there exists a hyperplane that separates two bricks of \( A \) completely.