Convexity

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Assignment Sheet 9

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Exercise 1
The Gamma function if defined as $\Gamma(x) = \int_0^\infty r^{x-1}e^{-r}\mathrm{d}r$ for $x > 0$. Prove that

1. $\Gamma(x+1) = x\Gamma(x)$.
2. $\Gamma(n) = (n-1)!$ for positive integer n.

Exercise 2
Prove the missing step in the computation of v_n, the volume of the ball B_1^n, from the lecture. Namely show that

$$\int_0^\infty nR^{n-1}e^{-\frac{R^2}{2}}\mathrm{d}R = 2^{\frac{n}{2}}\Gamma\left(\frac{n}{2}+1\right)$$

Exercise 3 [$\star$]
Let $A, B \subseteq \mathbb{R}^n$ be convex sets. Show the following equality of sets (in $\mathbb{R}^{n+1}$).

$$\operatorname{conv}\left((\{0\}\times A) \cup (\{1\}\times B)\right) \quad = \quad \bigcup_{t\in[0,1]} \left[\{t\}\times((1-t)A + tB)\right].$$

Exercise 4
Let $A \subseteq \mathbb{R}^n$ be a brick set consisting of at least two bricks. Show that there exist a canonic unit vector $e_i \in \mathbb{R}^n$ and $b \in \mathbb{R}$ and two bricks $B_1, B_2 \in A$ s.t. $e_i^\intercal x \leq b$ for all $x \in B_1$ and $e_i^\intercal x \geq b$ for all $x \in B_2$. That is, show there exists a hyperplane that separates two bricks of A completely.

The deadline for submitting solutions is **Thursday, December 8, 2016**.