Popular matchings

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The input is a graph $G = (V, E)$ where each vertex $v \in V$ ranks its neighbours in a strict order of preference. A matching $M$ is a set of edges no two of which share an endpoint.

For any two matchings $M$ and $M'$ we say that vertex $u$ prefers $M$ to $M'$ if $u$ is either matched in $M$ and unmatched in $M'$ or matched in both and prefers its partner in $M$ to its partner in $M'$. If $u$ is unmatched in both matchings, or matched to the same partner then we say that $u$ is indifferent between $M$ and $M'$.

Then given any two matchings $M$ and $M'$ we can have an election between them by having each vertex vote for the matching that it prefers. Vertices that are indifferent do not cast a vote. If $M'$ receives strictly more votes than $M$ we say that $M'$ won the election and $M$ lost the election.

Definition. A matching $M$ is popular if it does not lose an election against any other matching $M'$ of $G$.

It can be shown that every stable matching is a popular matching, meaning that popular matchings are a generalization of stable matchings. In particular a stable matching is a popular matching of minimum cardinality. Since a bipartite graph always admits a stable matching, popular matchings always exist in this case. Moreover, if $G$ is bipartite then there exists a polynomial time algorithm that finds a maximum cardinality popular matching [2].

In this project, we are interested in studying popular matchings in non-bipartite graphs. In particular we are interested in the complexity of the following open problem.

Open Problem 1. Let $G$ be a non-bipartite graph. Does $G$ admit a popular matching?

For non-bipartite graphs stable matchings do not always exists, and there is a polynomial-time algorithm that finds a stable matching whenever it exists. This leads us to the second open problem.

Open Problem 2. Let $G$ be a non-bipartite graph that admits a stable matching. Find a popular matching of maximum cardinality.

Note that given a matching $M$, one can test in polynomial-time whether $M$ is popular [1].

References
